Comment on "Confinement of Slave Particles in U(1) Gauge Theories of Strongly Interacting Electrons"

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In a recent Letter [1, 2], Nayak argued that the deconfinement can never happen in the U(1) gauge theory approach, because the "holon" charge q_b and "spinon" charge q_f can be assigned arbitrarily provided $q_b - q_f = e$ ($-e$ is the electron charge). This simple but important question seems never answered appropriately [3, 4]. In this Comment, I shall examine the issue more carefully.

Besides the dynamical gauge field a_μ, I introduce (smooth) external static gauge fields A^f_μ and A^b_μ which couple respectively to the spinon and holon fields. To obtain the low-energy effective action, high-energy parts of the dynamical fields are integrated out; their low-energy parts are kept fixed and are indistinguishable from the external fields. As a consequence, under A^f and A^b, the effective action must be written in terms of $\tilde{A}^\alpha_\mu \equiv A^\alpha_\mu + a_\mu$. Here I assume that the kinetic term of the gauge field is generated in the low energy effective action, as it is argued in the U(1) gauge theory approach. The low-energy effective Lagrangian density is thus written as

$$\begin{aligned}\mathcal{L} \;=\; & \mathcal{L}_f[\bar{f}_\sigma, f_\sigma, \tilde{A}^f_\mu] + \mathcal{L}_b[\bar{b}, b, \tilde{A}^b_\mu] + \frac{\lambda_f}{4}\tilde{F}^f_{\mu\nu}\tilde{F}^f_{\mu\nu} \\ & + \frac{\lambda_m}{2}\tilde{F}^f_{\mu\nu}\tilde{F}^b_{\mu\nu} + \frac{\lambda_b}{4}\tilde{F}^b_{\mu\nu}\tilde{F}^b_{\mu\nu} - a_\mu J^0_\mu, \qquad (1)\end{aligned}$$

where J^0_μ represents the constant density of one particle per site. Here $\tilde{F}^\alpha_{\mu\nu} = \partial_\mu \tilde{A}^\alpha_\nu - \partial_\nu \tilde{A}^\alpha_\mu$ ($\alpha = f, b$) and $\lambda_{f,m,b}$ are coupling constants. In the limit of zero external fields, we recover the usual form with the gauge field coupling constant $g^{-2} = \lambda_f + 2\lambda_m + \lambda_b$.

In the low-energy effective theory, one may consider the currents $J^{f,b}$ defined by the functional derivative of the action by the external fields $A^{f,b}$. The equation of motion of the dynamical gauge field a_μ leads to the "confinement constraint" $J^f_\mu + J^b_\mu = J^0_\mu$. However, these currents actually includes a contribution from the gauge field: $J^f_\mu = j^f_\mu + \lambda_f \partial_\nu \tilde{F}^f_{\mu\nu} + \lambda_m \partial_\nu \tilde{F}^b_{\mu\nu}$, where $j^f_\mu = \partial\mathcal{L}_f/\partial A^f_\mu$ is the spinon current is in a standard definition. The current j^b is similarly defined. The currents j^α do not obey the confinement constraint.

To discuss the charge of the particles, I set $A^f_\mu = q_f A_\mu, A^b_\mu = q_b A_\mu$, where A_μ is the physical electromagnetic vector potential which is regarded as an external field in this Comment. Starting from the tentative assignment $q_f = 0$ and $q_b = e$, the redefinition of the gauge field as $a_\mu \to a_\mu + ceA_\mu$ with an arbitrary parameter c induces the change in the charges [1, 2] $q_f = ce$ and $q_b = (c+1)e$. The physical current reads, after the redefinition, $J^{em}_\mu = ceJ^f_\mu + (1+c)eJ^b_\mu - ceJ^0_\mu$. This apparently depends on the arbitrary parameter c. However, the dependence on c turns out to be proportional to $J^f_\mu + J^b_\mu - J^0_\mu$, which vanishes thanks to the equation of motion. Thus the physical current is indeed independent of the charge assignment, as it should be.

What is the charge of a slave particle, if it is deconfined? For simplicity, below I take the limit of zero external field. The total physical current is given by $J^{em}_\mu = eJ^b_\mu = ej^b_\mu + e(\lambda_m + \lambda_b)\partial_\nu f_{\mu\nu}$, independent of c. Let us suppose that the spinon current j^f_μ is changed by δj^f_μ while j^b_μ is kept fixed. The change δj^f_μ affects the gauge field through the equation of motion, and the total change induced in the physical current reads $\delta J^{em}_\mu = -e\frac{\lambda_m+\lambda_b}{\lambda_f+2\lambda_m+\lambda_b}\delta j^f_\mu$. This means that, the effective charge of a deconfined spinon, *including the contributions from the gauge field*, is given by $Q_f = -e(\lambda_m+\lambda_b)/(\lambda_f + 2\lambda_m + \lambda_b)$ which is independent of the arbitrary parameter c of charge assignment. A similar analysis gives the effective charge of a deconfined holon Q_b, which satisfies $Q_b - Q_f = e$.

To conclude, the arbitrariness in the charge assignment observed by Nayak does not contradict with the possibility of the deconfinement. The effective charge carried by a deconfined slave particle appears to be fractional, which is independent of the arbitrary charge assignment but is rather determined dynamically. This fractional charge is apparently not quantized, unlike in the fractional quantum Hall effect. An estimate based on the one-loop calculation (see e.g. [5]) shows [6] that Q_b and Q_f are proportional to $(1-x)$ and x respectively, where x is the doping concentration. This implies that the "holon" indeed carries most (but not all!) of the charge near half filling ($x \ll 1$). More reliable non-perturbative calculations of the effective charges Q_α, along with the examinations on whether the deconfinement really takes the place, would be desired.

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[6] I thank P. A. Lee for a discussion leading to this.